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TOPICAL ISSUE

Extended Lanchester–Osipov Model for Accounting Combat Units with Single Burst Effect in Strategic Computer Games

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Abstract—A model has been constructed that describes the confrontation between two armies, each of which simultaneously contains two types of combat units: with continuous and discrete fire. The structure of the optimal army composition, compiled as a response to the known composition of the enemy army, has been studied. To test the theory, simulations of battles were carried out in a simple strategic game — autobattler.

Keywords: operations research, strategic games, Lanchester–Osipov models

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1. INTRODUCTION

Lanchester–Osipov models, designed to describe the dynamics of military confrontations, arose more than a century ago [1, 2]. In the initially proposed version, they rely on systems of differential equations that describe the depletion of resources of the opposing sides. The specific form of the equations is determined from the conditions in which the battle takes place.

In some cases, it turns out that it is more convenient to describe the system under consideration using discrete models, for example, this is typical for naval battles [3–5].

This paper considers a synthetic discrete-continuous case, typical of modern strategic computer games (Real-Time Strategy/RTS) [6–8], in which players often need to form armies that simultaneously contain combat units with quasi-continuous fire (such as infantry, infantry fighting vehicles, tanks, etc., potentially capable of firing many shots during one battle), and with single-shot fire (for example, kamikaze drones, this also includes many types of artillery, the effect of which during the battle, significantly limited or even once). Creating artificial intelligence for games of this kind is one of the most pressing and complex machine learning problems today [8, 9].

In setting up a specific task, as well as in the experimental part, we started from the mechanics of autobattlers [10, 11] — a type of cellular strategy games in which battles take place automatically, and the gameplay is focused on compiling an optimal set of combat units and their spatial location. However, similar approaches can be applied in the future to analyze battles in arbitrary other subgenres of strategy games.

2. MODEL

In modern strategy games, each combat unit usually has the following basic characteristics:

— Health reserve. Successful enemy attacks reduce the target's health, and when the supply is completely depleted, the unit dies.

— Damage per second (DPS) is the average damage taken from the target in one successful attack, divided by the time between two consecutive attacks.

— Attack radius is the maximum distance to the target at which a combat unit can attack.

— Special abilities, such as dealing significant damage to a single enemy or group of enemies within a certain area once. Using the ability either requires the expenditure of special resources (the energy of a given unit), or has a long delay (cooldown) before being used again.

To describe the battles of armies consisting of combat units without special abilities, we will use a recently proposed model [6], which relies on the dynamics of depletion of the total health of two armies instead of depletion of their numbers. This approach allows, for example, to explain why, when controlling identical, even homogeneous, armies, a more experienced player will statistically win over a novice. For heterogeneous armies, this model, as confirmed in the experiment, demonstrates a significant improvement in the accuracy of predicting the outcome of battles taking place in automatic control mode compared to the classical Lanchester–Osipov models.

In this work, we consider the expansion of this model, which is necessary when including combat units with the simplest special ability that has a one-time effect in the army. The following section outlines the characteristics of the specific types of units discussed in this article.

2.1. Characteristics of Unit Types

We are considering a minimalistic version of the autobattler, in which there are only three types of combat units: **swordsmen, archers** and **mages**, the cost of production of each combat unit is assumed to be the same (the designations themselves are introduced for convenience, and could be replaced if desired , for example, on *tanks, helicopters, submarines*). In a typical autobattler there are many different subtypes of units in each of these categories, but in this work we do not seek to solve the problem in the most general form, but to explore the basic properties of such systems. The following describes the characteristics of each type of combat unit:

— **Swordsman** has 1 health and DPS, i.e. health and DPS are normalized to the characteristics of a swordsman.

— **Archer** has $h < 1$ health and $d > h^{-1}$ damage, so in a 1 v 1 confrontation the archer destroys the swordsman. At the same time, in a complex army, swordsmen act as "human shields" for archers [6], i.e., the condition for the possibility of causing damage to the archers of army A is the destruction of all swordsmen in army A.

— **Mage** has the ability to inflict b damage to the enemy before the start of the battle, while $h < b < 1$, i.e., in a 1 v 1 battle mage will destroy an archer, but not a swordsman. The mage does not perform any other functions except the initial infliction of damage, and against one swordsman one mage is defeated. We suppose that mages can deal damage to enemy archers before the battle, and if there are no archers left, then the damage is dealt to swordsmen.

Thus, these three combat units are lined up in a "rock-paper-scissors" configuration. This case is interesting because game balance is achieved here, that is, it is impossible to give deliberate preference to any of the presented types of units when composing an army.

In this work, we will be interested in the optimal response to a given enemy army, i.e., optimal composition of units with a fixed total cost with a known composition of enemy units. The optimality criterion is maximizing the Lanchester strength of the army surviving at the end of the battle. The next subsection discusses the Lanchester strength of a mage-free army.

2.2. Basic Model

In the absence of mages, we have a standard system, which can be described by the basic model [6]. Let there be $s \geqslant 0$ swordsmen and $a \geqslant 0$ archers, $s + a = 1$. The strength of their composition is determined by the expression

$$S = \int\limits_0^{ah+s} dy \int\limits_0^y \rho(x)dx, \tag{1}$$

here ρ is the distribution of DPS density, characterizing the decrease in the total DPS of the army when receiving damage, $ah + s$ is the total health of the army at the beginning of the battle. In this work we use the simplest approximation, which has the form

$$\rho(x) = d/h + (1 - d/h)\theta(x - ah), \tag{2}$$

where θ is the Heaviside function. Substituting (2) into (1), we obtain an expression for the strength of composition:

$$S = a^2 dh/2 + a(1 - a)d + (1 - a)^2/2. \tag{3}$$

The optimal composition in this case does not depend on the composition of the enemy army. From $dS/da = 0$ we get

$$a^* = \frac{d - 1}{d(2 - h) - 1}. \tag{4}$$

For $dh > 1$ we obtain $a^* > 0.5$, i.e. There are always more archers needed than swordsmen (equality is achieved on the curve $dh = 1$).

The maximum strength is equal to

$$S^* = \frac{d(d - h)}{2d(2 - h) - 2}. \tag{5}$$

Let's discuss some properties of the expression (5). For $h = 1$, the optimal army, according to (4), consists of only archers and has a strength of $S^* = d/2$. At $h \ll 1$ the maximum force is $S^* \approx d^2/(4d - 2) \approx d/4$ (and according to our conditions $d > h^{-1} \gg 1$). For $h = d^{-1} + \delta_h$, where $\delta_h \ll 1$ we obtain $S^* \approx (d + 1)/4 + d\delta_h/8$.

Note also that if we have G gold, then the optimal proportion of units is preserved, and the strength of the army scales quadratically, $S^*_G = G^2 S^*$. Having analyzed the problem of balancing an army in the absence of mages, we move on to the general case in the next subsection.

2.3. General Case

Consider the effect of enemy mages on armies. Let's denote the numbers of swordsmen, archers and mages in our army as s_1, a_1, m_1 and for the enemy army as s_{-1}, a_{-1}, m_{-1}. The Hamiltonian (difference of strengths) can be written as follows:

$$H = S_1 - S_{-1} = \int\limits_{\xi_1}^{a_1 h + s_1} dy \int\limits_{\xi_1}^{y} \rho_1(x)dx - \int\limits_{\xi_{-1}}^{a_{-1} h + s_{-1}} dy \int\limits_{\xi_{-1}}^{y} \rho_{-1}(x)dx, \tag{6}$$

where $\xi_i = \min(m_{-i}b, a_i h + s_i)$ is the amount of health taken from the army by enemy mages at the beginning of the battle.

First, consider the case $m_1 = 0$, $a_1 + s_1 = 1$, $m_{-1} \neq 0$, i.e., when we need to create an optimal composition of only swordsmen and archers, taking into account the presence of enemy mages. It is easy to see that in this case there are two fundamental strategies: either not produce archers at all, in which case all the damage from the mages will be taken by the swordsmen, but only swordsmen will participate in the battle, or produce more archers than the enemy mages will burn.

When choosing to produce only swordsmen, the final strength of the army has the form

$$S^*_{1s} = (1 - m_{-1}b)^2/2. \tag{7}$$

Here $m_{-1}b$ means the amount of gold burned by enemy mages.

When choosing to produce archers, enemy mages will cause more monetary damage $m_{-1}b/h$, but in this case the remaining money can be spent on creating an optimal composition with a share of archers (4) and strength

$$S^*_{1a} = S^*(1 - m_{-1}b/h)^2\theta(1 - m_{-1}b/h). \tag{8}$$

The optimal sthength is equal to the maximum of these two:

$$S^*_1 = \max\left(S^*_{1s}, S^*_{1a}\right). \tag{9}$$

At $m_{-1} = 0$, obviously, archers should be produced, and at $m_{-1}b \geqslant h$ the damage of enemy mages is greater than the health of the archers produced with all the money, accordingly, there is no point in producing them. The effectiveness of strategies is equal at the point

$$m^*_{-1} = \frac{h(\sqrt{2S^*} - 1)}{b(\sqrt{2S^*} - h)}. \tag{10}$$

Thus, the minimum number of enemy mages at which it becomes profitable to refuse to produce archers is inversely proportional to the damage of mages, and depends in a complex way on the parameters of archers d, h included in the expression (5). Also note that $2S^* > 1 > b > h > 0$, which implies $1 > m^*_{-1} > 0$.

Let's move on to the case when $m_1 \neq 0$, $a_1 + s_1 + m_1 = 1$. Both strategies — producing archers and abandoning them — now need to be balanced with producing your own mages. At the same time, the mages produced affect both the strength of one's own army (by reducing the amount of money that can be spent on it) and the strength of the enemy's by causing damage to it. The difference in forces can be written for both cases in the form

$$S_1 - S_{-1} = (G - m_1)^2 S^+_1 - (a_{-1} - m_1 b/h)^2 dh/2 - (a_{-1} - m_1 b/h)s_{-1}d - s^2_{-1}, \tag{11}$$

where G is the amount of money remaining after the attack of enemy mages (depending on our strategy) and $S^+_1 = 0.5$ when refusing to produce archers, $S^+_1 = S^*$ when choosing to produce archers. The expression (11) is correct in the region $0 < m_1 < a_{-1}h/b$. From $\partial H/\partial m_1 = 0$ we obtain

$$m^+_1 = \frac{2GS^+_1 h - db(a_{-1}h + s_{-1})}{2S^+_1 h - db^2}. \tag{12}$$

The final expression for the optimal number of mages has the form

$$m^*_1 = \min(m^+_1, a_{-1}h/b). \tag{13}$$

To find the final answer, you need to substitute the values of G and S^+_1 for both strategies in (12) and further in (11), compare them and choose the best strategy. Qualitatively, the situation is similar to that discussed earlier in the absence of the ability to produce mages, i.e., when the number of enemy mages is small, you need to produce archers, when there is a large number, you need to refuse, however, in the general case, the expression for the switching point between strategies, similar to (10), turns out to be quite cumbersome and we will not give it. The next section contains the results of numerical calculations of the Hamiltonian for various parameters, as well as experiments in a game simulator.

3. SIMULATIONS AND DISCUSSION

Figure 1a shows the dependence of the Hamiltonian (6) on the proportion of units in army 1 with a fixed composition of army 2, consisting of the same number of all types of combat units. In this case, with these parameters (indicated in the caption to the figure), the strategy of refusing to produce archers dominates, and the optimal share of mages is $m_1 = a_2h/b = 2/9$. The figure also shows the least profitable strategy — producing archers with total health equal to the damage of enemy mages, and refusing to produce your own mages.

Figure 1b shows the dependence of the number of survivors in army 1 when composing the optimal answer on the composition of enemy army 2. As can be seen from the figure, in this case, for a player who does not have an information advantage and the ability to compose an optimal answer, it is possible to minimize the damage from the opponent's information advantage by choosing an appropriate strategy that at least maximizes the amount of damage caused to the opponent. And if a player with an informational advantage has an economic lag, then this strategy will minimize the economically stronger player's own losses.

As a further development of this topic, we could consider optimal disinformation — communicating certain information to the enemy in order to produce an optimal response that has the most favorable appearance for the disinforming party. Let us recall that in the absence of combat units with a single action, there were no such problems (or, on the other hand, opportunities) at all, because the optimal composition (4) did not depend on the composition of the enemy army, and the player with the most money won. In this same task, the information factor becomes a multiplier, effectively increasing the combat potential by tens of percent, see in Fig. 1b the spread in the range of 20–50%. Thus, with comparable economic and technological capabilities of the players, the information factor becomes critically important.

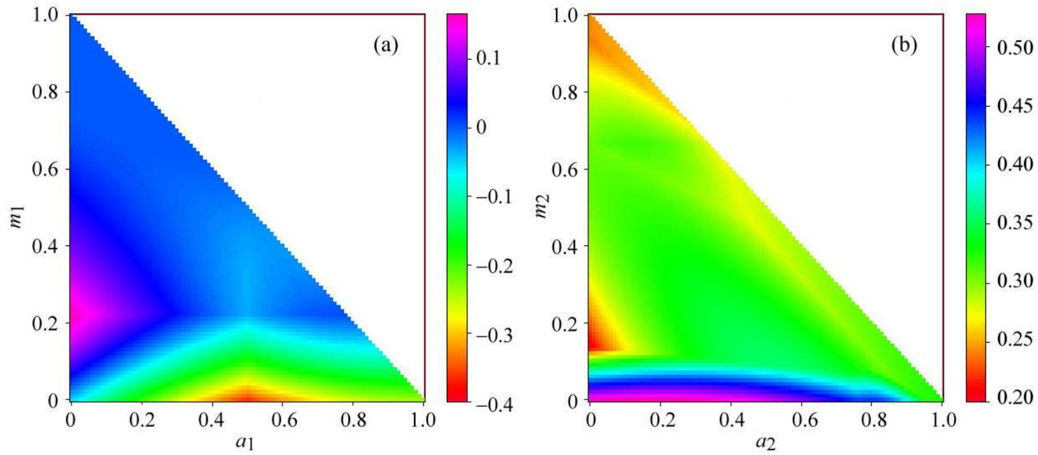

Fig. 1. (a) Dependence of the Hamiltonian (6) on m_1, a_1 for $m_2 = s_2 = a_2 = 1/3$ and $s_1 = 1 - m_1 - a_1$. Parameters of combat units $d = 3$, $h = 0.5$, $b = 0.75$. (b) Dependence of the benefit of information advantage (the ratio of the cost of the surviving army with the optimal response to the initial total cost of the army) on the parameters of the enemy army m_2, a_2 for $s_2 = 1 - a_2 - m_2$.

3.1. Simulations in an Autobattler

Continuous Lanchester–Osipov models are, by their nature, designed to describe large-scale battles. In this work, we conducted experiments to test the applicability of our model to describe the dynamics of battles of small groups of units.

For numerical simulation, a minimalistic autobattler was created — an environment in which the battle takes place on a cellular field, and each combat unit occupies exactly 1 cell. The battle occurs as follows: for each unit, a target is searched for from enemy units within the attack range. Of these, the one is selected whose quickest destruction will weaken the enemy army the most, that is, with the highest ratio of DPS to current health. As with most autobattlers, if a unit already has a target, it does not change it until the target is destroyed. After selecting targets, the act of skirmishing occurs where units deal damage to their targets and units that run out of health disappear. After this, units that did not attack this turn due to the lack of targets within the damage radius take a step towards the center of the enemy army.

The simulation was carried out on a field measuring 20×10. The initial arrangement of units in each implementation was made randomly with the following conditions: blue team archers can appear in the first three columns, then blue team swordsmen appear in the next four columns, after which there is a neutral territory with a width of 6, and the situation is mirrored. An example of the initial deployment of armies is shown in Fig. 2.

According to standard autobattler mechanics, eight players are at the table and take turns fighting each other. With each defeat, the loser is awarded a number of penalty points equal to the number of surviving enemy units. When a player reaches a critical number of penalty points, he is eliminated from the game. The game ends when one player remains. We neglected to consider the economic component of the autobattler, but considered the number of survivors as a result of the battle as an objective function.

Simulations were carried out for three army compositions of player 2:

1) 5 swordsmen, 5 archers, 0 mages;
2) 4 swordsmen, 3 archers, 3 mages;
3) 1 swordsman, 2 archers, 7 mages.

For the armies of player 1, all possible combinations of units were tried with a fixed sum of 10. For each composition, 21 experiments were carried out with different random initial deployments of both armies and the results were averaged (if the blue player won, the surviving units were considered negative numbers, and if the red player won, they were considered positive).

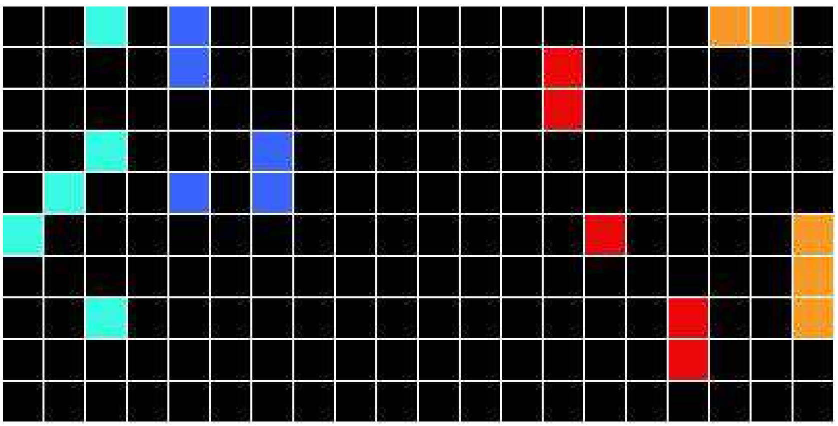

Fig. 2. An example of the initial arrangement of two armies in an auto battler, both consisting of five swordsmen and five archers. Blue and red colors indicate swordsmen, blue and orange — archers. The field has a size of 20×10.

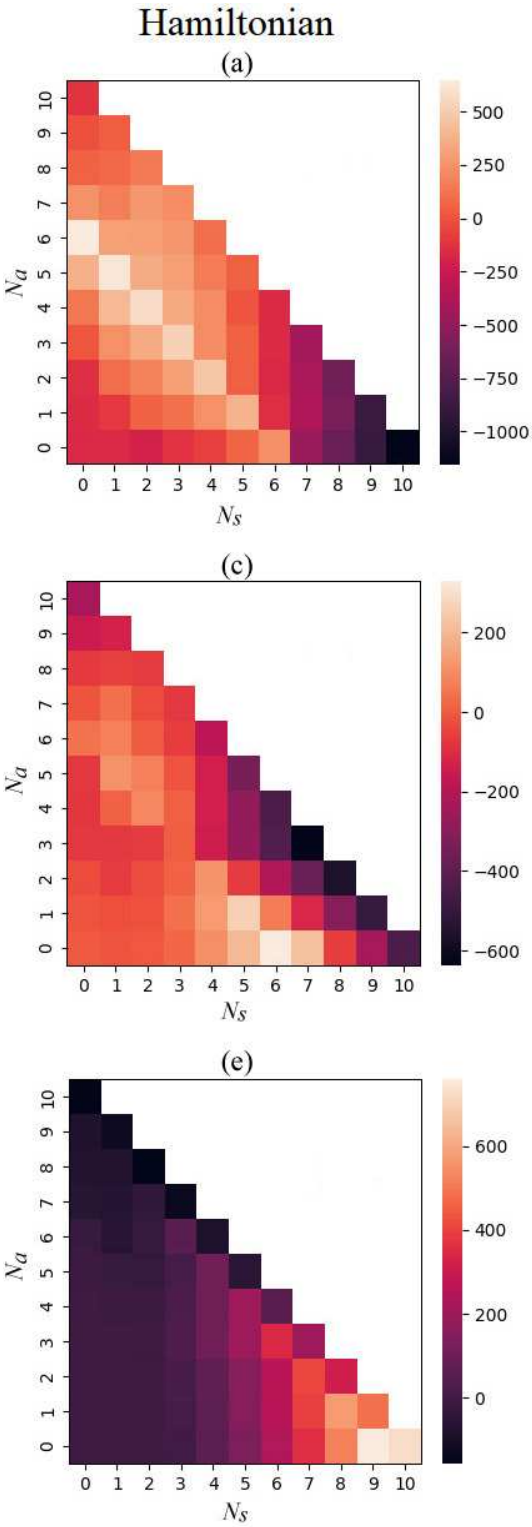

Fig. 3. Dependences (a, c, e) of the Hamiltonian of the system (b, d, f) of the results of numerical experiments to measure the average number of survivors from the composition of army 1. The army of player 2 consists of (a, b) 5 swordsmen, 5 archers, 0 mages (c, d) 4 swordsmen, 3 archers, 3 mages; (d, f) 1 swordsman, 2 archers, 7 mages. Along the axes: N_s is the number of swordsmen in the army 1, N_a is the number of archers, the number of mages is $10 - N_s - N_a$. Swordsmen parameters: 60 health, 0.5 damage per turn, attack radius 3 cells; The archer has 20 health, 2 damage per turn, attack radius 4.5 cells; The mage's initial damage is 25 health points.

In Fig. 3 the results of numerical experiments (b, d, f) are compared with the corresponding Hamiltonians (a, c, e) in the same color schemes. In all three cases considered, the optimal composition predicted by our Hamiltonian coincides with that in the game. Also, the similarity of the distribution patterns as a whole is visible to the naked eye. Thus, even in an essentially discrete system, our model, which does not take into account the spatial location and speed of movement of combat units, still gives good predictions of battle statistics and optimal compositions in this sense. The problem of strict optimization of army composition in an autobattler is NP-complete [12], however, we have built a fairly accurate approximation that can be used to solve practical problems.

4. CONCLUSION

We proposed a mathematical model that describes the dynamics of the battle of armies containing simultaneously combat units with continuous fire and single-action special abilities. The case of having a choice of three types of combat units, related to each other like "rock-paper-scissors", is analyzed in detail, one of which has a one-time effect, and the other two have continuous fire. Analytical expressions are obtained for the composition of the optimal response to a known enemy army in this case. It is shown that the developed discrete-continuous model is well applicable not only to the description of large-scale battles, but also for battles of small groups of about 10 combat units in autobattlers.

We analyzed not only ways to use the information advantage — through drawing up an optimal response to the enemy army — but also ways to defend ourselves in conditions where the player suspects that the enemy has such an advantage, but he himself does not have information about the enemy army. For subsequent work, approaches to optimal disinformation of the enemy are also of great interest, namely, providing him with such false information about one's army, the optimal response to which would be most beneficial to the disinforming player.

In this work, we limited ourselves to considering the symmetrical case (two players have the same amount of money and the same characteristics of combat units), this is a typical situation for computer games, however, in practice, strategies may also be interesting for cases of significant advantage, technological and/or economic, of one of the players. The mechanics of action of combat units with a single effect that we have considered are also not the only possible ones, and in the future, research may be carried out on a wide variety of other mechanics of a single action.

Also the development of similar approaches for spatial models of battles, similar to those outlined in the works [13, 14], seems promising.

From the point of view of practical applicability, the strategies and approaches discussed in this work can be useful for study by military and political experts. Wargames with similar mechanics serve as one of the important analytical tools in geopolitics, many key centers, such as RAND and CSIS, have a rich history of producing such games and using them to predict geopolitical scenarios [15–17].

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